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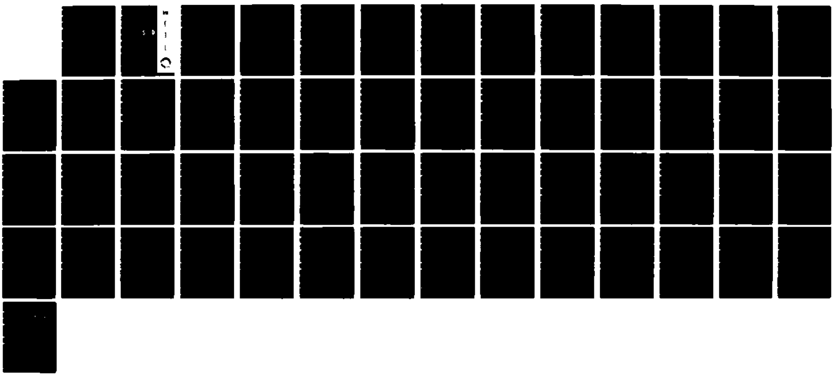

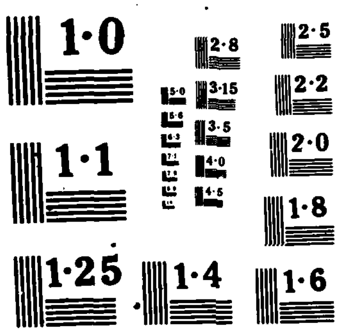

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Techniques to improve astronomic positioning in the field

Angel A. Baldini

December 1985

Prepared for
U.S. ARMY CORPS OF ENGINEERS
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This paper deals with new methods and techniques for improving astronomic positioning in the field. Latitude and longitude are obtained by observing transit times of pairs of stars over a fixed vertical plane, independent of azimuth and zenith distances. A unique solution is derived for each pair.

Higher accuracy in latitude can be obtained by observing transit times of star pairs over the prime vertical, where the parallactic angle reaches its -

maximum value. The vertical plane of observation can be fixed within 90 arc seconds with respect to the prime vertical without changes in the star's parallactic angle, and a function of it, the latitude, can then be computed. The star transit times over different vertical lines are thereby reduced to the central line or collimation plane, as a function of the parallactic angle.

Higher accuracy in longitude can be achieved by observing the transit times of pairs of stars over a vertical plane fixed within 20 arc minutes with respect to the meridian plane. Each individual star pair will determine a solution. Since each pair does not depend on azimuth orientation, the star pairs can be chosen arbitrarily with respect to declination or zenith distance, and short periods of clear sky observations can be utilized. When several pairs are observed an adjustment can be carried out through the equations of conditions that allow one to detect errors in either the transit times or in the star's right ascensions.

Preface

This study was conducted under DA Project 4A161102B52C, Task A, Work Unit 00003.

The study was done under the supervision of Dr. H. G. Baussus Von Luetzon, Team Leader, Center for Geodesy; and Dr. Robert D. Leighty, Director, Research Institute.

COL Alan L. Laubscher, CE was Commander and Director; and Mr. Walter E. Boge was Technical Director of the Engineer Topopgraphic Laboratories during the report preparation.

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TECHNIQUES TO IMPROVE ASTRONOMIC POSITIONING IN THE FIELD

INTRODUCTION

This report is a sequel to the paper "Techniques for the Improvement of Astronomic Positioning in the Field," presented at the ASP-ACSM Convention, Washington D.C.[1]

Based on Taylor's theorem, formulas were derived to reduce the observed transit times of a star over several vertical lines, the intervals of which are unknown, when the instrument is reversed between the star transit or when the instrument is kept in a fixed position. The influence of instrumental errors upon the transit times and the corrections upon them is also achieved. Simulation field data observations were used for testing and evaluating the equations shown in this report. The tests showed a complete agreement between theory and practical work. A comparison of the methods of Niethammer and Struve was made with the author's method. Results of the test conducted on site at the Goddard Space Flight Center are also included.

BACKGROUND

Previously, the author has derived two types of equations to obtain latitude by observing the transit times of a star pair on a vertical plane fixed within 2 minutes of arc with respect to the prime vertical.[2] The first equation gives the latitude from

$$\tan \phi = \pm \frac{\sqrt{\cos^2 P + \tan^2 \delta}}{\sin P} \quad \begin{matrix} + \text{ west} \\ - \text{ east} \end{matrix} \tag{1}$$

where P represents the parallactic angle of a star, computed from the equations

$$\tan x = \frac{\cos \frac{1}{2}(\delta_w - \delta_e)}{\sin \frac{1}{2}(\delta_w + \delta_e)} \cot \tfrac{1}{2}\sigma \tag{2}$$

---

[1] Angel A. Baldini, "Techniques for the Improvement of Astronomic Positioning in the Field;" presented at the ASP-ACSM Convention, Washington, D.C., 23-27 February 1981.

[2] Ibid.

$$\tan y = \frac{\sin \frac{1}{2}(\delta_w - \delta_e)}{\cos \frac{1}{2}(\delta_w + \delta_e) \cot \frac{1}{2}\sigma}$$

where

$$\sigma = (Tw - Te)\ (1+\mu+C) - (\alpha_w - \alpha_e) + d\sigma_w + d\sigma_e \tag{3}$$

and the parallactic angles are obtained from

$$\begin{aligned} P_w &= x + y \\ P_e &= 360 - x + y \end{aligned} \tag{4}$$

where μ is the clock rate, C is a constant, the value of which is $C = 0$ when the interval of time $(T_w - T_e)$ is sidereal time, and $C = 0.002737908$ if the interval of time corresponds to mean time.

The second equation gives the latitude from

$$\tan \phi = \frac{\sin t}{\cos \delta \tan P} + \cos t \tan \delta \tag{5}$$

With limitations, equation (5) can be used in any vertical plane. The parallactic angle may be derived from a star pair, either equation (2) or

$$\tan P_w = \frac{\sin \sigma}{\cos\delta_w \tan\delta_e - \sin\delta_w \cos \sigma}$$

$$\tan P_e = \frac{\sin \sigma}{\sin \delta_e \cos\sigma - \cos\sigma_e \tan\delta_w} \tag{6}$$

INVESTIGATION

Error in Latitude Resulting from Errors in Transit Times. First consider equation (1). Differentiating this equation with respect to σ, we have

$$d\phi = \frac{\partial \phi}{\partial P_w} \frac{\partial P_w}{\partial \sigma}$$

finding that

$$\frac{\partial \phi}{\partial P} = -\frac{1}{\tan P_w \tan \phi}$$

$$\frac{\partial P}{\partial \sigma} = -\frac{\cos P_e \sin P_w}{\sin \sigma}$$

hence

$$d\phi = +\frac{\cos P_e \cos P_w}{\tan \phi \sin \sigma} d\sigma \tag{7}$$

Similarly, from equation (5) we have obtained, considering the west star of the pair,

$$d\phi = \frac{\cos \phi}{\tan A_w} dt_w - \frac{\cos \phi \sin t_w \cos P_e}{\sin A_w \sin \sigma} d\sigma \tag{8}$$

Equation (1) has been derived under the condition that a star pair is observed in a fixed vertical plane, which must be within ±2 minutes of arc with respect to the prime vertical. On the other hand, equation (5) does not have this limitation with respect to the vertical plane of observation, but an error dt upon the hour angle increases as the angle increases between the plane of observation and the prime vertical, as can be seen by examining the first term on the right hand side or equation (8). When the angle of separation is 12'. that is $90^\circ \pm 12'$, an error dt = 1^s, gives an error upon latitude

$$d\phi < 0\overset{''}{.}05 \cos \phi \tag{9}$$

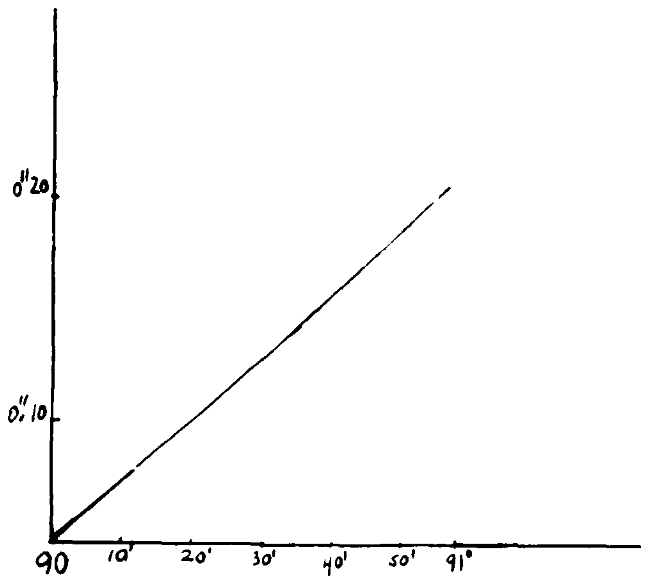

Figure 1. Error for latitude of 40 degrees.

Figure 1 gives the error that one can expect on a latitude of 40 degrees, assuming an error of $dt = 1^s$.

The expression for the error in ϕ resulting from an error $d\sigma$ is analyzed.

Error in Latitude Resulting From Errors in Transit Times. The following errors affect $d\sigma$:

1. The instrumental error, resulting from an error in determining the collimation and inclination of the rotation axis.

2. The personal error, occurring from the psycho-physical characteristics of the observer.

3. The displacement error, arising from an atmospheric displacement of the star.

A suitable, well-chosen observation procedure can reduce the influence of instrumental error. Using a self-recording micrometer, frequently called the impersonal micrometer, can reduce a great deal of the personal error. The error arising from unsteadiness of the star must not be taken because displacement of the star occurs vertically.

We may form an estimate of the total effect of all the errors by examining the several values of the transit times reduced to a common central line.

Reducing The Observed Transit Times of a Star, the Interval of Threads Being Unknown. We proceed to investigate the formula for reducing the observations on the side threads to the middle thread. Let

$\theta_1, \theta_2 =$ the star's transit times over the same vertical line with respect to the direct and reverse instrument position.

Consider these transit times as a function of the star's azimuths, and let

$A_1, A_2 =$ the star's azimuth that corresponds to the times θ_1 and θ_2, respectively.

Let A_o be the mean azimuth

$$a_o = \frac{A_1 + A_2}{2}$$

and let θ_o be the transit time over the vertical line whose azimuth is A_o. The transit times are functions of the azimuth. We, therefore,

may write, according to the Taylor's theorem

$$\theta_i = f(Ai) = f(Ao + \Delta Ai) =$$

$$\theta_i = \theta_o + \frac{d\theta}{dA} \Delta Ai + \frac{1}{2} \frac{d^2\theta}{dA^2} \overline{\Delta A}^2 + \frac{1}{6} \frac{d^3\theta}{dA^3} \overline{\Delta A}_i^3$$

where

$$\Delta A_i = A_i - A_o$$

For the star's transit times over the same vertical line

$$i = 1,2$$

we have

$$\Delta A_1 = \frac{A_1 - A_2}{2}$$

$$\Delta A_2 = \frac{A_2 - A_1}{2}, \tag{11}$$

and for the mean of the two transit times, we obtain according to equation (10)

$$\frac{\theta_1 + \theta_2}{2} = \theta_o + \frac{1}{2} \frac{d^2\theta}{dA^2} (A_2 - A_1)^2, \tag{12}$$

Therefore, the mean transit time is obtained from

$$\theta_o = \frac{\theta_1 + \theta_2}{2} - \frac{1}{8} (A_2 - A_1)^2 \frac{d^2\theta}{dA^2} \tag{13}$$

The star's hour angle rate with respect to azimuth is related to the parallactic angle P and its zenith distance Z through the equation

$$\frac{dt}{dA} \cos\delta \cos P = \sin Z \tag{14}$$

Differentiating this equation with respect to A, we have

$$\frac{d^2t}{dA^2} \cos\delta \cos P - \frac{dt}{dA} \cos\delta \sin P \frac{dP}{dA} = \cos Z \frac{dZ}{dA} \tag{15}$$

In the prime vertical

$$\left(\frac{dP}{dA}\right)_{A \pm 90} = 0 \tag{16}$$

From equation (14) and the equation

$$\frac{dZ}{dt} = \cos \delta \sin P \tag{17}$$

we get

$$\frac{dZ}{dA} = \tan P \sin Z \tag{18}$$

Because we insert equations (16) and (18) into equation (15) and because the sidereal time θ is related to the hour angle and right ascension through the equation

$$\theta = t - \alpha, \tag{19}$$

we have

$$\frac{d^2\theta}{d^1A^2} = \frac{d^2t}{dA^2}, \tag{20}$$

Whereby in the prime vertical, we obtain

$$\left(\frac{d^2\theta}{dA^2}\right)_{A = \pm 90} = \frac{\cos Z}{\cos \delta \cos P} \tan P \sin Z, \tag{21}$$

from which equation (13) becomes

$$\theta_o = \frac{\theta_1+\theta_2}{2} - \frac{1}{8} \frac{\cos Z}{\cos \delta \cos P} \tan P \sin Z (A_2-A_1).^2 \tag{22}$$

The zenith distance is evaluated from the equation

$$- \cos \phi \cos A = \sin \delta \sin Z - \cos \delta \cos Z \cos P$$

which for $A = \pm 90^o$, gives

$$\tan z = \cos P \cot \delta \tag{23}$$

To evaluate $(A_1 - A_2)$, proceed as follows. Consider the azimuth as a function of time. Applying Taylor's theorem, we have

$$a_1 = A_o + \frac{dA}{dt} \tau_i + 1/2 \frac{d^2A}{dt^2} \tau_i^2 + \frac{1}{6} \frac{d^3A}{dt^3} \tau_i^3 + \cdots \tag{24}$$

where

$$\tau_i = \theta_i - \theta_o$$

Evaluating A_O for the time

$$\theta_o = 1/2\,(\theta_1 + \theta_2),$$

the two values of τ_i that are to be used in equation (24) are

$$\tau_1 = \frac{\theta_1 - \theta_2}{2}$$

$$\tau_2 = \frac{\theta_2 - \theta_1}{2}$$

By substraction between the two equations so derived, we obtain that

$$A_2 - A_1 = \frac{dA}{dt}(\theta_2 - \theta_1) + \frac{1}{48}\frac{d^3A}{dt^3}(\theta_2 - \theta_1)^3. \qquad (25)$$

The second term on the right side of equation (25) is always small and may be disconsidered when squaring, keeping with full accuracy that

$$(A_2 - A_1)^2 = \left(\frac{dA}{dt}\right)^2 (\theta_2 - \theta_1)^2. \qquad (26)$$

In the prime vertical we have

$$\left(\frac{dA}{dt}\right)_{A = \pm 90^0} = \sin\phi,$$

but also

$$\cos z = \frac{\sin\delta}{\sin\phi},$$

through which we found that

$$(A_2 - A_1)^2 = \sin^2\delta / \cos^2 Z. \qquad (27)$$

Inserting this value into equation (22) and remembering equation (23), we finally get for computing the time θ_o,

$$\theta_o = \frac{\theta_1+\theta_2}{2} - \frac{1}{8} \sin \delta \tan P \, (\theta_2-\theta_1)^2. \tag{28}$$

Expressing $(\theta_2-\theta_1)$ in degrees and a fraction of the degree, and indicating by $\delta\theta$ the correction to the mean time, we have

$$\delta\theta = 0^s.5236 \sin \delta \tan P[(\theta_2-\theta_1)]^2. \tag{29}$$

Hence, the time θ_o is now reduced to

$$\theta_o = \frac{\theta_1+\theta_2}{2} \pm \delta\theta \tag{30}$$

The correction $\delta\theta$ has a positive sign for one of the stars of the pair and a negative value for the second star. The sign to be applied is known from the fact that the $(\theta_1+\theta_2)/2$ values have a tendency to increase or decrease from the first transit time to the center transit time. The sign is positive if $(\theta_1+\theta_2)/2$ increases, and negative if it decreases.

Influence of Instrumental Errors Upon $\delta\theta$. In obtaining the correction $\delta\theta$ it was assumed that no instrumental errors exist. But they do. Let us consider them and their influence upon $\delta\theta$. Two of the more important errors to be considered are the collimation error and the inclination error of the horizontal axis of rotation. Consider first the collimation error.

Influence of the Collimation Error Upon $\delta\theta$. We observe the star transit times on all vertical lines on one side of the middle line, then reverse the instrument 180° and observe the star on the same line on the opposite side of the middle line. By this mode of observation, the same line is alternately towards north and towards south and at precisely the same distance from the collimation axis. It is evident, therefore, that by this mode of observation the effect of collimation error cancels out in the sum $(\theta_1+\theta_2)/2$, but does not vanish in the difference $(\theta_2-\theta_1)$. On the contrary, it affects it twice, including consequently in the evaluation of $\delta\theta$.

Let c be the collimation error, and let $\bar{\theta}_1$, $\bar{\theta}_2$ be the star transit times over the same thread before and after the instrument has been rotated 180^o in azimuth. We assume observations with theodolite as Wild T-4, DKA3, etc.

Let P_1 be the parallactic angle that corresponds to the first observation, so the correction time to θ_1 is given by

$$\delta\theta_1 = c \sec \delta \sec P_1 \tag{31}$$

and therefore the star transit time must be

$$\theta_1 = \bar{\theta}_1 + c \sec \delta \sec P. \tag{32}$$

When the instrument is rotated about the vertical axis 180^o, the collimation changes sign. Then the transit time over the same thread on the other side with respect to the central one, the collimation correction has the opposite sign. The correction to the transit time is

$$d\theta_2 = - c \sec \delta \sec P,$$

and the real transit time is therefore

$$\theta_2 = \bar{\theta}_2 - c \sec \delta \sec P \tag{33}$$

It can be seen that the sum of the transit times over the same thread is

$$\theta_1+\theta_2 = \bar{\theta}_1+\bar{\theta}_2$$

but the difference of these transit times is affected twice owing to the collimation error. Consequently, we have

$$\theta_2-\theta_1 = \bar{\theta}_2-\bar{\theta}_1 - 2 c \sec P \sec \delta \tag{34}$$

which is the value to be used in parenthesis of equation (29). Consider now the influence of inclination error.

Influence of Inclination Error Upon the Transit Times and on $\delta\theta$. The inclination error affects the transit time in the amount

$$d\theta_i = i \cos z \sec \delta \sec P, \tag{35}$$

which in the prime vertical reduces to

$$d\theta_i = i \tan \delta / \cos P \sin \phi \tag{36}$$

Let

i_1 = inclination of the first observation
i_2 = inclination of the second observation

Then, for the sum of the transit times, we have

$$\theta_2+\theta_1 = \bar{\theta}_1+\bar{\theta}_2 \pm (i_2+i_1) \tan \delta \sec P \operatorname{cosec} \phi, \tag{37}$$

and for the difference of the transit times, we obtain

$$\theta_2-\theta_1 = (\bar{\theta}_2-\bar{\theta}_1) \pm (i_2-i_1) \tan \delta \sec P \operatorname{cosec} \phi. \tag{38}$$

The ± sign shall be identified later.

Total Influence of Instrumental Errors. The total influence of collimation and inclination errors on the mean of the two observed times on each side thread is

$$\theta_2+\theta_1 = \bar{\theta}_2+\bar{\theta}_1 \pm (i_2+i_1) \tan \delta \sec P \sec \phi, \tag{39}$$

and on the differences between the observed times is

$$\theta_2-\theta_1 = \bar{\theta}_2-\bar{\theta}_1 \pm 2c \sec P \delta \pm (i_2-i_1) \tan \delta \sec P \operatorname{cosec} \phi. \tag{40}$$

The correct sign to be considered is shown later.

Equations (39) and (40) must be considered when using the instrument alternately in opposite positions of the rotation axis, reversing it between observations of the same star. It should be noted that if there is no error in the observed times, the value of the time θ_o, which is computed for each thread, should be equal one to each other.

An important factor has to be considered. The latitude is determined by a pair of stars, one at east of the meridian and the other at west of the meridian. It is assumed that the azimuth of the instrument has not changed during the observations. This is an important factor when a Wild T-4 or similar instrument is used. The Wild T-4 theodolite has no mechanism for lifting and turning the horizontal axis on its bearings, as the Bamberg and Askania universal telescopes have, which are favorites for this kind of observation. This type of instrument has a reversing apparatus enabling the observer to turn the transit quickly from the direct to the reverse position. In the Wild T-4 or Kern DKM 3A, the observer must rely on the horizontal scale readings and is therefore subjected to azimuth errors in setting up the instrument orientation. For this type of instrument, one pair or more must be observed with the theodolite fixed in a direct position, recording the star transit times for all threads, then reversing the instrument and setting it up through the horizontal scale reading close to 180^{o} with respect to the first position, and making new pairs of stars observations. It is to be noted that in this case equations (39) and (40) must be modified since they were derived on the condition that the star transits are on all the threads on one side of the middle thread. Then, by reversing the instrument 180^{o}, the star transits are on the same threads on the opposite side of the middle thread. Let us now consider the reduction of the observed transit times when the instrument is not reversed during the observation of one star pair.

Reduction of the Observed Transit Times of a Star When the Instrument is Not Reversed. Let n be the number of threads and let θ_n be the star transit times. By construction, the threads are located almost symmetrically with respect to the middle thread. Consider the transit times over threads 1 and n, 2 and n-1, 3 and n-3 and so on, so the times to be considered are θ_1 and θ_n; θ_2 and θ_{n-1}; θ_3 and θ_{n-3}, and so on.

Let

i_1 = inclination before observation

i_n = inclination after observation

c = collimation error

$\bar{\theta}_1$, $\bar{\theta}_2$ = observed transit times

The total influence of inclination and collimation errors is for either $\bar{\theta}_1$ or $\bar{\theta}_n$;

$$d\theta = \frac{1}{2}(i_1+i_2) \tan\delta \sec P \operatorname{cosec}\phi + c \sec\delta \sec P \qquad (41)$$

Therefore, we have for the sum and on the difference of times,

$$\theta_1+\theta_n = \bar{\theta}_1+\bar{\theta}_n - 2\,\delta\theta \qquad (42)$$

Then the reduced time θ_o becomes

$$\theta_o = \frac{\bar{\theta}_1+\bar{\theta}_n}{2} \pm \frac{(i_1+i_n)}{2} \frac{\tan\delta}{\cos P \sin\phi} \pm \frac{c}{\cos P \cos\delta} \qquad (43)$$

$$- 0^s\!.524 \sin\delta \tan P \,[(\theta_n-\theta_1)^o]^2$$

A similar equation holds for every two symmetrical threads. It is to be noted that the second and third terms in the right side of equation (33) are constants, that the collimation changes its sign when observations are made in reverse instrument position, and that the term in parenthesis $(\bar{\theta}_n - \bar{\theta}_1)$ must be taken in degrees and fractions of the degree.

The central thread has a value of

$$\theta_o = \bar{\theta}_o \pm \frac{1}{2}(i_1+i_n) \frac{\tan\delta}{\cos P \sin\phi} \pm \frac{c}{\cos\delta \cos P} \qquad (44)$$

It is to be noted that in all equations derived so far, nothing is mentioned about the sign to be considered for the corrections due to collimation and inclination errors. Let us consider now when a positive or negative sign should be used.

Sign to be Applied for the Corrections Due to Inclination Errors. Equations (31) and (36) were considered without analysis of the sign to be applied to the observed transit times. Let us analyze the inclination error. Consider the instrument brought to the prime vertical. Then, consider that the rotation axis is perpendicular to the plane of the prime vertical and lies in the intersection of the planes of the meridian and horizon.

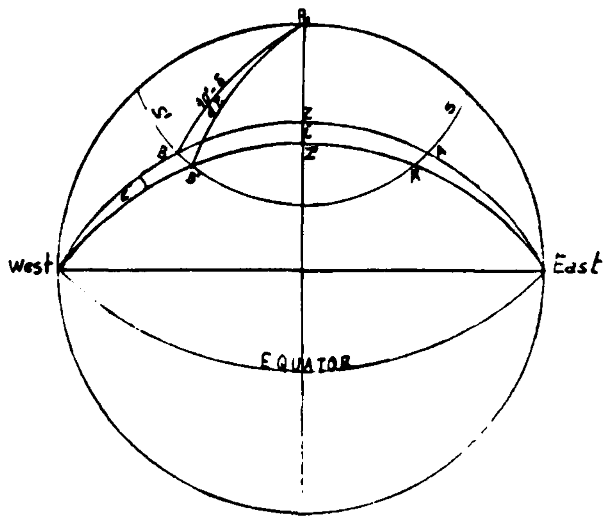

Figure 2. Influence of the Inclination Error.

Let $P_n Z$, figure 2, be the meridian, WZE the prime vertical of the observer, and SS^1 the parallel of a star that crosses the meridian between the zenith and the equator. Such star crosses the prime vertical at A and B. When no error of inclination exists, the rotation axis is horizontal and lies in the meridian plane; then, the telescope will describe a vertical circle WZE.

Let us assume that the north end of the axis is above the plane of the horizon. Let i be the angle inclination. The telescope, then, will describe the circle WZ^1E, where Z' is the vertical of the instrument; therefore, ZZ' = i. A star is observed at either A' or B'. Consider the west observation. The star is observed at B^1, instead of being observed at B. Let t_1 be the hour angle when the star is observed at B' and t + dt be the hour angle when the star is observed at A. Let θ^1 be the observed time when the star is observed at B', and θ when the star is observed at A.

From the known relation

$$t = \theta' - \alpha$$
$$t + dt = \theta - \alpha,$$

we have

$$dt = \theta - \theta',$$

therefore

$$\theta = \theta' + dt,$$

which by using equation (36) becomes

$$\theta_w = \theta'_w + i_w \tan \delta_w \sec P_w \operatorname{cosec} \phi. \qquad (45)$$

We feel also that the east observation takes place after the star crosses the prime vertical. Therefore, we must substract to the observed time $\bar{\theta}'$ at A^1 for the amount of time the star moves on its parallel, from B to B^e. A similar equation to (45) for the east transit time is

$$\theta_e = \theta'_e - i_e \tan \delta_e \sec P_e \operatorname{cosec} \phi. \tag{46}$$

It follows, then, that the correction due to inclination error when the north end of the rotation axis is higher than the south must be positive to the west observation and negative to the east observation. To fulfill this condition, the inclination must have the same sign as the coefficient of i has. The inclination is determined through the position of the bubble on a graduated scale on the surface of the glass tube of the stride level. We must then consider what sign is to be given when the zero of the scale position is towards the north or towards the south.

To obtain the inclination of the axis of rotation, we must read the bubble position with respect to the etched scale on the tube with the level in direct and reverse position. Let NS, figure 3, be the axis of rotation with the north end over the horizon HH', forming an angle of inclination i. When the level is on the straight line NS, the bubble always moves to the highest point of the tube A or B, figures 3 and 4 respectively.

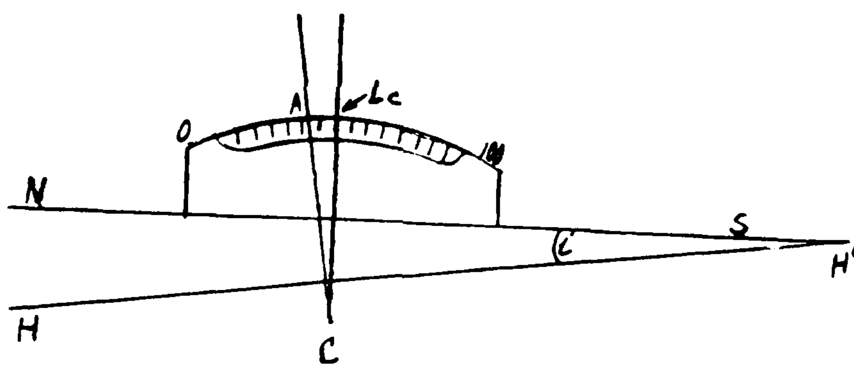

Figure 3. Bubble Position when Zero of the Scale is Towards the North.

first position, and $1/2\,(\ell'_n + \ell'_s)$ from the second position. According to figures 3 and 4, we have

$$L_c > 1/2\,(\ell_n+\ell_s) \qquad \text{First position}$$

$$L_c < 1/2\,(\ell'_n+\ell'_s) \qquad \text{Second position}$$

The center point L_c has the value

$$L_c = 1/4\,[(\ell_n+\ell_s) + (\ell'_n+\ell'_s)]. \tag{47}$$

The inclination value is

$$\begin{aligned} i &= \rho\,[\,L_c - 1/2\,(\ell_n+\ell_s)] && \text{First position} \\ i &= \rho\,[\,1/2\,(\ell'_n + \ell'_s) - L_c] && \text{Second position} \end{aligned} \tag{48}$$

The mean value gives

$$i = \frac{\rho}{4}\left[(\ell'_n + \ell'_s) - (\ell_n - \ell_s)\right] \tag{49}$$

Let us analyze the sign that corresponds to the bubble position. Equation (36) gives the correction to the transit by

$$d\theta_i = i \tan\delta \sec P \operatorname{cosec}\phi \tag{50}$$

Since the coefficient of i,

$$\tan\delta \sec P \operatorname{cosec}\phi > 0$$

the sign of i to be used in equation (49) must be positive. As

$$(\ell^1_n+\ell^1_s) > (\ell_n+\ell_s)$$

we consider positive the bubble readings when the zero of the scale is towards the south, and negative towards the north. Adding algebraically the two readings and dividing by four, the inclination is obtained with

the correct sign, as can be seen from this example. The following readings of the level were considered:

Zero south	-19	-74	=	-93
Zero north	+39	+94	=	133
		sum		+40

Inclination = +10 divisions

The positive sign indicates that the north support of the instrument is higher than the south. In this case it is 10 divisions for both the direct and reversed positions.

Consider now the influence in the transit times due to a collimation error.

Sign to be Applied for the Corrections Due to Collimation Errors. Let c be the collimation constant of a thread at north of the collimation axis in the prime vertical. The small circle of the sphere, which corresponds to it, is south of the prime vertical.

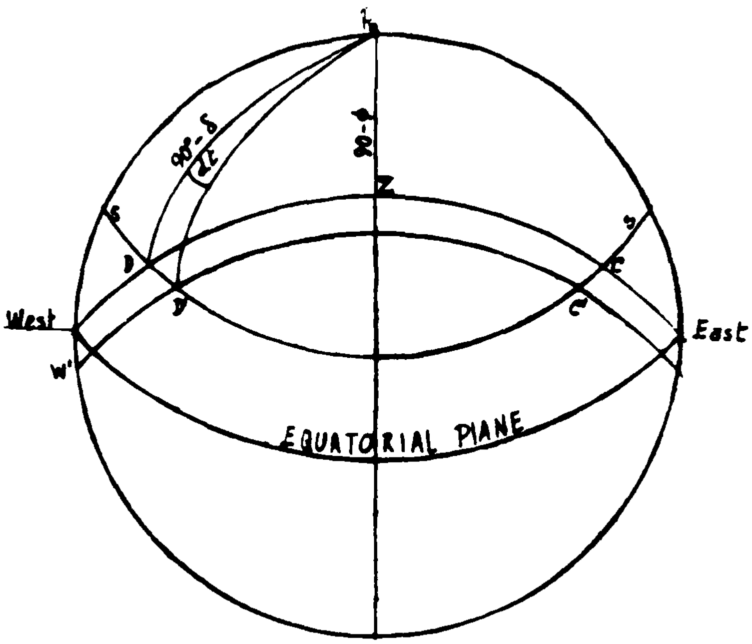

Figure 5. The Influence of Collimation Error on a Star Transit Over the Prime Vertical.

Let SDC, figure 4, be the diurnal circle of a star. Because of the collimation error, the telescope will not describe the great circle of arc WDZC, but the smallest W'D'Z'C' circle of arc. The star's transit times refer to the star position over C and D, but owing to collimation, the transit times will at C' and D'. It can be seen from figure 4 that the east transit time is after crossing C, and the west transit is observed

before the star crosses the prime vertical. Therefore, the east observation requires a negative correction, and the west a positive correction.

The correction owing to collimation is given by the following equation:

$$d\theta = c \sec \delta \sec P \tag{51}$$

It follows then that the transit times of a star on the prime vertical will be

$$\begin{aligned} &\text{West} \qquad \theta_w = \bar{\theta}_w + c \sec \delta_s \sec P_w \\ &\text{East} \qquad \theta_e = \bar{\theta}_e - c \sec \delta_e \sec P_e \end{aligned} \tag{52}$$

The sign that corresponds to these equations, and also to equations (45) and (46), indicates what sign is to be applied in the preceding equations in which a ± sign appears.

The plus sign corresponds to a star observed west of the prime vertical, and the negative sign to a star observed east of the vertical.

Determination of the Collimation Error. To determine the collimation error, we may use a well-defined object as distant as possible, near the horizon, and read the horizontal circle in direct and reverse instrument position, deducing the collimation, c, from the following equation:

$$c = \tfrac{1}{2}(A_2 - 180 - A_1)$$

where A_1, A_2 are the horizontal readings in direct and reverse theodolite position. In field work, an object that is well-defined and at a sufficient distance cannot always be found. In this case, a star of high declination can be observed following the procedure indicated in reference 1. A third procedure may be considered by using a second theodolite, by focusing it to infinite, and by pointing the two theodolites precisely one to other. Illuminating the second theodolite from the rear, the crosswires are projected in the T-4, on the focal plane, like a signal at infinity. Then, carry out the determination of the collimation error as it is indicated above, in the first case.

Examples of Computation. Examples of computations included in this report are as follows:

1. Simulation field data observation was used for testing and evaluating the theory developed for determining latitude from transit times of a star pair over a vertical plane within 2 minutes of arc with respect to the prime vertical. The test conducted showed a complete agreement between theory and practical work.

2. The purpose of this example was to compare Niethammer's method against Baldini's method. The test conducted is based on data taken from Niethammer.[3] The test conducted shows a discrepancy of 0.06 second of arc in the latitude. The Baldini equations are less complicated to apply than those of Niethammer, and they are independent of the Niethammer requirements mentioned above.

3. The example included is a test that involves potential sources of error in latitude determination by selecting star pairs according to ocular instrument position. Large discrepancies have been found, depending on the pair selection. The value obtained when the pairs are formed using (N-S) and (S-N) or (S-N) and (N-S) are different from those obtained using (N-S) and (N-S) or (S-N) and)S-N).

4. In this example a comparison is made between Baldini's method against Struve's method. Data used is from Niethammer. The test shows a discrepancy of 0.15 second of arc.

5. Results of testing Baldini's method at the Goddard Space Flight Center, Greenbelt, MD.

Example Using Simulation Data. Star transit times for a west and east star forming a pair are shown in table 5. The first column indicates the vertical line number, the second column indicates the star transit times in the direct instrument position, and the third column indicates the transit times when the instrument was reversed. Similarity is for the star at east. Below each column the inclination and collimation values are given. The data refer to a place whose latitude is 40° north.

Latitude is computed from the equation

$$\tan \phi = \frac{\sqrt{\cos^2 P + \tan^2 \delta}}{\sin P}$$

[3] T. Niethammer, 1947 - Die Genauen Methoden Der Astronomisch - Geographischen Ortbestimmung, Verlag Birkhauser Basel.

The parallactic angle P is computed through the following equations:

$$\tan \tfrac{1}{2}(A+B) = \frac{\cos \tfrac{1}{2}(\delta_w - \delta_e)}{\sin \tfrac{1}{2}(\delta_w + \delta_e)} \cot \tfrac{1}{2}\sigma_{12}$$

$$\tan \tfrac{1}{2}(A-B) = \frac{\sin \tfrac{1}{2}(\delta_w - \delta_e)}{\cos \tfrac{1}{2}(\delta_w + \delta_e)} \cot \tfrac{1}{2}\sigma_{12}$$

$$\tfrac{1}{2}(A+B) = x \qquad P_w = x+y = A$$

$$\tfrac{1}{2}(A-B) = y \qquad P_e = 360 - x+y = -B$$

Table 1. Star Transit Times

Number of Observations	West Star Direct	West Star Reverse	East Star Direct	East Star Reverse
1	$10^h\ 04^m\ 49^s.169$	$10^h\ 07^m\ 34^s.039$	10 21 34.104	10 24 17.362
2	4 56.600	26.528	41.590	9.917
3	5 2.794	20.272	47.828	24 3.711
4	8.990	14.018	54.063	23 57.504
5	12.092	10.893	21 57.181	54.400
6	15.192	7.767	22 0.298	51.296
7	21.395	7 1.519	6.531	45.086
8	27.601	6 55.274	12.763	38.875
9	30.705	52.152	15.879	35.769
10	10 05 33.809	10 6 49.031	10 22 18.994	10 23 32.663
	$i = +1''.50$	$i = +5''.50$	$i = +2''.50$	$i = -1''.20$
	$c = +3''.11$	$c = -3''.11$	$c = +3''.11$	$c = -3''.11$

$\alpha_w = 6^h\ 00^m\ 00^s.00$ $\qquad\qquad$ $\alpha_e = 13^h\ 40^m\ 00^s.00$

$\delta_w = 30^0\ 00\ 00''.00$ $\qquad\qquad$ $\delta_e = 20^0\ 00'\ 00''.00$

i = inclination

c = collimation constant

Preliminary evaluation of the parallactic angles and zenith distances.

Constants

$$C_1 = \frac{\cos \frac{1}{2}(\delta_w - \delta_e)}{\sin \frac{1}{2}(\delta_w + \delta_e)} \qquad\qquad C_2 = \frac{\sin \frac{1}{2}(\delta_w - \delta_e)}{\cos \frac{1}{2}(\delta_w + \delta_e)}$$

$$C_1 = 2.3571\ 97472 \qquad C_2 = 0.0961\ 65772$$

$$\tan \tfrac{1}{2}(A+B) = C_1 \cdot \cot \tfrac{1}{2}\sigma^{12} \qquad P_w = A$$

$$\tan \tfrac{1}{2}(A-B) = C_2 \cdot \cos \tfrac{1}{2}\sigma^{12} \qquad P_e = -B$$

The values of the parallactic angles P_w and P_e may be found accurately enough from the mean transit times of the last observations. Thus, we have

$$\theta_w = 10^h\ 06^m\ 11\overset{s}{.}42 \qquad \alpha_w = 13^h\ 40^m\ 00\overset{s}{.}00$$

$$\theta_e = \underline{10\quad 22\quad 55.83} \qquad \alpha_e = \underline{6\quad 00\quad 00.00}$$

$$\theta_w - \theta_e \quad - 0^h\ 16^m\ 44\overset{s}{.}41 \qquad \alpha_w - \alpha_e = 7^h\ 40^m\ 00\overset{s}{.}00$$

$$\sigma_{12} = (\theta_w - \theta_e) + (\alpha_e - \alpha_w) = 7^h\ 23^m\ 15\overset{s}{.}59$$

$$\sigma_{12} = 110^0\ \ 48'\ \ 53\overset{''}{.}85 \qquad \tfrac{1}{2}\sigma_{12} = 55^0\ \ 24'\ \ 26\overset{''}{.}92$$

$$\tan \tfrac{1}{2}(A+B) = 1.6256\ 6752 \qquad \tfrac{1}{2}(A+B) = 58^0 24'\ \ 10\overset{''}{.}8$$

$$\tan \tfrac{1}{2}(A-B) = 0.0663\ 2176 \qquad \tfrac{1}{2}(A-B) = 3\quad 47\quad 39.8$$

$$P_w = A = 62^0 11' 50\overset{''}{.}6$$

$$P_e = -B = -54\quad 36\quad 31.0$$

The values of the zenith distances are obtained from

$$\tan Z = \cos p \cot \delta$$

$$\tan Z_w = 0.8078\ 7487 \qquad Z_w = 38^0 56' 01.9$$

$$\tan Z_e = 1.5912\ 2578 \qquad Z_e = 57\quad 51\quad 10.2$$

With these values of P_w, Z_w, P_e and Z_e, the inclination and collimation corrections were evaluated through the equations

$$I = \frac{i \cos Z}{15 \cos \delta \cos P} \qquad \Upsilon = \frac{c}{15 \cos \delta \cos P}$$

They are shown on table 2. The factor 15 is introduced to obtain these corrections in time.

Table 2. Inclination and Collimation Errors

Star	I_1	I_2	γ_1	γ_2	I_2-I_1	γ_2-I_1	$\frac{1}{2}(I_2+I_1)$
West	$0^s.193$	$0^s.706$	$+0^s.513$	$-0^s.513$	$-1^s.027$	$-1^s.027$	$0^s.450$
East	0.163	-0.078	+0.381	-0.381	-0.241	-0.762	+0.042

Hence, the evaluation of the transit times over the central vertical line is

$$\tau = \theta_2 - \theta_1 \pm (I_2 - I_1) \pm (\gamma_2 - \gamma_1) \quad \pm \begin{matrix} \text{West} \\ \text{East} \end{matrix}$$

$$\delta\theta = - 0^s.5256 \tan P \sin \delta \, (\tau^0)^2$$

$$\theta_o = \frac{\theta_1 + \theta_2}{2} \pm \frac{I_2 + I_1}{2} - \delta\theta$$

τ^0 is expressed in degrees

The computation of the time θ_o was carried out according to equation (34), which is shown in tables 3 and 4 for the west and east star, respectively.

Table 3. Reduction of the observed transit times - west star

$\frac{1}{2}(\theta_1 + \theta_2)$	$\tau = \theta_2 - \theta_1$	$\delta\theta$	$\theta_o = \frac{1}{2}(\theta_1+\theta_2) - \delta\theta + \frac{1}{2}(I_1+I_2)$
$10^h\ 06^m\ 11^s.604$	$2^m\ 44^s.356$	$0^s.233$	$10^h\ 06^m\ 11^s.821$
11.564	2 29.414	0.192	11.822
11.533	2 16.964	0.163	11.821
11.505	2 04.514	0.134	11.821
11.492	1 58.287	0.121	11.821
11.480	1 52.061	0.108	11.822
11.438	1 27.159	0.066	11.822
11.428	1 20.933	0.056	11.822
10 06 11.420	1 14.708	0.048	10 06 11.822

Table 4. Reduction of the observed transit times - east star

$\frac{1}{2}(\theta_2+\theta_1)$	$\tau = \theta_2 - \theta_1$	$\delta\theta$	$\theta_o = \frac{1}{2}(\theta_1+\theta_2) - \delta\theta - \frac{1}{2}(I_1+I_2)$
$10^h\ 22^m\ 55^s.733$	$2^m\ 44^s.261$	$-0^s.116$	$10^h\ 22^m\ 55^s.808$
55.754	2 28.327	- .096	55.808
55.769	2 15.884	-0.081	55.808
55.784	2 03.441	- .067	55.809
55.790	1 57.219	- .060	55.808
55.797	1 50.998	- .054	55.809
55.808	1 38.555	- .043	55.809
55.819	1 26.112	- .032	55.809
55.824	1 19.890	- .028	55.809
10 22 55.829	1 13.669	- .024	10 22 55.810

With the θ_0 transit times of the west and east star, we obtain σ_{12} as follows:

$$\theta_{o_w} - \theta_{o_e} = -\ 0^h\ 16^m\ 43^s.988$$

$$\alpha_e - \alpha_w = \underline{7\quad 40\quad 00.000}$$

$$\sigma_{12} = 7\quad 23\quad 16.012 = 110^0\ 49'00''.18$$

$$1/2\,\sigma = 55^0\ 24'30.09$$

Hence, with the already known constants C_1 and C_2, we now get the parallactic angles through the equations

$$\tan 1/2\,(A+B) = C_1 \cot \sigma_{12} \cdots\cdot\ 1.6256\ 14150$$

$$\tan 1/2\,(A+B) = C_2 \cot \sigma_{12} \cdots\cdot\ 0.0663\ 19585$$

$1/2\,(A+B) = 58^0\ 24'\ 07''.78$ $\qquad P_w = 62^0\ 11'\ 47''.72$

$1/2\,(A-B) = 3\quad 47\quad 39.79$ $\qquad P_e = -54\quad 36\quad 28.38$

and the latitude from the equation

$$\tan \phi = \frac{\sqrt{\cos^2 p + \tan^2 \delta}}{\pm \sin P} \quad \pm \begin{matrix}\text{west}\\ \text{east}\end{matrix}$$

west star $\quad \phi = 40^0 00'\ 00.00$
east star $\quad \phi = 40\ 00\ \ 00.00$

Comparison Between Niethammer's Method Against Baldini's Method.

For this comparison, the same field observational data taken from Niethammer was considered.[4] The computation shown here is based on Baldini's equation of using a star parallactic angle,

$$\tan \phi = \frac{\pm\sqrt{\cos^2 P + \tan^2\delta}}{\sin P} \quad \pm \begin{matrix}\text{west}\\ \text{east}\end{matrix}$$

[4]T. Niethammer, 1947. Die Genauen Methoden Der Astronomisch. Geographischen Ortbestimmung, Verlag Birkhuaser Basel.

Niethammer's method of latitude computation requires three preconditions.

1. The clock correction must be exactly known.

2. The instrument must be brought to within a few seconds of an arc with respect to the prime vertical.

3. The azimuth of the northern end of the rotation axis must also be known.

Extra field work is then necessary to fulfill these three requirements.

Since Niethammer's final equation,

$$\tan \phi = \tan p_w \cos (t_w - \mu_n),$$

is exactly the equation of a star transit over the prime vertical,

$$\tan \phi = \tan \delta / \cos t_v,$$

where

$$\phi = 90^\circ - \Phi$$

$$\delta = 90^\circ - p_w$$

$$t_v = t_w - \mu_n.$$

Therefore, an error in the clock correction affects the angle t_w. An error of the northern orientation of the rotation axis affects the value μ_n. The total effects over the latitude are

$$d\phi = - \frac{\tan \phi}{(1+\tan^2\phi)} \tan (t_w - \mu_n)(dt_w - d\mu_n).$$

Baldini's equation is independent of items 1 and 3. With respect to item 2, only a rough approximation within 2 minutes of arc is required.

In the example herein considered, Niethammer gives the latitude evaluated from the mean-reduced transit times. Baldini's computation gives also the latitude for each individual event as shown on table 4.

It can be seen that for a maximum difference in the angle σ between event 1 and 11,

$$d\sigma = 12''42$$

gives a discrepancy in the latitude

$$d\phi = 1''14$$

This value agrees with the equation of error shown at the bottom of table 4.

Computation Using Niethammer's Data. The second column of table 5 contains the mean values of the transit times of the west star α Cyg. The third column gives the difference between the transit times for the same vertical line for the direct and reverse instrument position. The fourth column contains the corrections to the mean time event computed according to the equation

$$\delta\theta = 0\overset{s}{.}5236 \sin \delta \tan P \left[(\theta_2-\theta_1)^{o}\right]^2$$

The mean value θ_m increases from event 1 to 11 and the corresponding corrections $\delta\theta$ decreases; therefore, the corrections $\delta\theta$ must be added to the mean values θ_m, which are shown in column 5.

Table 5. Star: α Cyg West

Even	$\theta_m = \frac{1}{2}(\theta_1+\theta_2)$	$\theta_2-\theta_1$	$\delta\theta$	$\theta_o = \theta_m + \delta\theta$
1	$22^h\ 13^m$ 43.11	3^m 19.62	$0\overset{s}{.}833$	$22^h\ 13^m$ 43.943
2	43.29	3 09.82	.753	44.043
3	43.48	2 59.16	.671	44.151
4	43.50	2 49.64	.601	44.111
5	43.40	2 37.92	.521	43.921
6	43.83	2 29.50	.467	44.297
7	44.00	2 19.64	.408	44.408
8	43.92	2 09.06	.348	44.268
9	43.90	1 59.24	.297	44.197
10	44.20	1 48.06	.244	44.444
11	44.59	1 36.44	.194	44.784
Mean	$22^h\ 13^m\ 43\overset{s}{.}747$		$0\overset{s}{.}485$	$22^h\ 13^m\ 44\overset{s}{.}232$

Table 6, second column, shows the mean transit times of the east star λ Andr. In this case the mean time θ_m decreases from event 1 to 11 and the $\delta\theta$ decreases; therefore, $\delta\theta$ must be substracted from the mean time. It is shown in column 5.

Table 6. Star: λ Andr. East

Even	$\theta_m = \frac{1}{2}(\theta_1 + \theta_2)$	$\theta_2 - \theta_1$	$\delta\theta$	$\theta_o = \theta_m - \delta\theta$
1	$22^h\ 24^m\ 09^s.34$	$3^m\ 53^s.28$	$1^s.543$	$22^h\ 24^m$ 07.797
2	08.87	3 40.54	1.379	07.491
3	08.76	3 26.84	1.213	07.547
4	08.81	3 14.02	1.067	07.743
5	08.54	3 00.32	0.922	07.610
6	08.31	2 46.02	0.782	07.528
7	08.32	2 33.00	0.664	07.656
8	07.98	2 20.70	0.561	07.419
9	07.94	2 08.08	0.465	07.475
10	08.36	1 52.12	0.356	08.004
11	08.09	1 39.36	0.280	07.810
Mean	$22^h\ 24^m$ 08.484		0.839	$22^h\ 24^m\ 07^s.645$

Tables 3 and 4 are related to the process-computing latitude. The coefficient K is the mean width of contact, and i is the inclination; the values of which are

$$K = +\ 0^s.047$$

$$i = +\ 0''.82$$

Table 7. Determination of latitude

		West	East
1	θ_o	$22^h\ 13^m\ 44^s.232$	$22^h\ 24^m\ 07^s.645$
2	$\delta\theta$	$+0.227$	$+0.301$
3	1+2	$22^h\ 13^m\ 44^s.459$	22 24 07.946
4	α	$20^h\ 39^m\ 24^s.570$	23 34 41.100
5	δ	$45^0\ 04'\ 29''.10$	$46^0\ 08'\ 34''.33$
6	3-4	$1^h\ 34^m\ 19^s.889$	$-1^h\ 10^m\ 33^s.154$
7	$\bar{\sigma}_{12}$	$2^h\ 44^m\ 53.043$	
8	$\bar{\sigma}^o_{12}$	$41^0\ 13'\ 15''.64$	
9	$1/2\ \sigma$	$20^0\ 36'\ 37''.82$	
10	$1/2\ d\sigma i$	$+4.47$	
11	9+10	$20^0\ 36'\ 42''.29$	
12	$1/2\ (\delta_w - \delta_e)$	$-0^0\ 32'\ 02''.61$	
13	$1/2\ (\delta_w + \delta_e)$	$45^0\ 36'\ 31''.71$	

$$\cos Z = \sin \delta / \sin \phi o$$

$$\delta\theta = K \frac{\sin Z}{\sin \phi - \cos Z \sin \delta}$$

$$d\sigma_i = i\left(\frac{\cot Z_w + \cot Z_e}{\sin \delta}\right)$$

$$\tan 1/2\,(A+B) = \frac{\cos\ (12)}{\sin\ 13} \cot\ (11)$$

$$\tan 1/2\,(A-B) = \frac{\sin\ (12)}{\cos\ 13} \cot\ (11)$$

$$P_w = A$$

$$P_e = 360-B$$

$$\tan \phi = \pm \sqrt{\frac{\cos^2 A + \tan^2 \delta}{\sin A}}$$

+ west star
− east star

$\tan 1/2\,(A+B) = 3.7206\ 26378$ $\qquad (A+B) = 74^0 57'\ 21''.70$

$\tan 1/2\,(A-B) = -0.0354\ 26216$ $\qquad 1/2\,(A-B) = -2^0\ 01'\ 44''.13$

$$A = 72^0 55'\ 37''.57$$

$$\phi = 47^0 32'\ 27''.39$$

Table 8. Evaluation of latitude for each event

Event	σ	$P_w = A$	Latitude	$\phi-\phi_m$
1	41°13' 17".97	72°55'39"73	47°32'26".68	-0".64
2	24.06	37.74	27.24	-0.08
3	24.86	37.48	27.31	-0.01
4	21.30	38.64	26.99	-0.33
5	20.44	38.92	26.91	-0.42
6	27.32	36.68	27.54	+0.22
7	27.06	36.76	27.52	+0.20
8	28.52	36.28	27.65	+0.33
9	26.61	36.90	27.48	+0.16
10	25.38	37.31	27.36	+0.04
11	41 13 30.39	72 55 35.67	47 32 27.82	+0.50

$$\phi_m = 47^{0}32'\ 27''32 \pm 0''.10$$

Niethammer gives $\phi = 47^{0}32'27''.28$, showing a discrepancy of 0"04.[5]

Niethammer's method requires that the clock correction must be known; the azimuth of the northern end of the rotation axis must also be known; and the instrument must be set up within a few seconds of arc with respect to the prime vertical. The only requirement of Baldini's method is to set up the instrument within 2 minutes of arc with respect to the prime vertical.

Example Using Star Pairs According to Ocular Instrument Position. To analyze how the selection of star pair influences the latitude determination, consider the following procedure.

We observe the star transit times on all the vertical lines on one side of the middle vertical line, then reverse the instrument 180°, and observe again the star on the same vertical lines on the opposite side of the middle vertical line. Therefore, if the instrument position has ocular north in the first part, the second part has ocular south, and the second star of the pair is observed in the reversed mode: first, with

[5]Niethammer. op. cit.

ocular south and the second with ocular north. Data used for this investigation are also taken from Niethammer.[6]

The first star pair considered has ocular position: N-S for the west star and S-N for the second star. A second pair is formed: N-S for the east star and S-N for the second star. The latitude values so obtained show a large discrepancy as can be seen in the table below:

Table 9. Latitude values

Pair	Star West	Star South	Latitude
1-2	N-S	S-N	47^0 32′ 29.″47
3-4	S-N	N-S	47 32 26.08
	Same stars but in reversing selection order		
1-3	N-S	N-S	47^0 32′ 27.″64
2-4	S-N	S-N	47 32 27.76

The formula used for this investigation is a function of the parallactic angle:

$$\tan \phi = \frac{\pm \sqrt{\cos^2 P + \tan^2 \delta}}{\sin P} \quad \begin{matrix} + \text{ west} \\ - \text{ east} \end{matrix}$$

To verify the above finding, a second test was conducted. A new formula was derived that is a function of the star's hour angles.

$$\tan \phi = \frac{\tan \delta_e \sin t_w - \tan \delta_w \sin t_e}{\sin \sigma}$$

$$\sigma = (\theta_w - \theta_e) - (\alpha_w - \alpha_e)$$

This formula is valid for any vertical plane; but for the observation of star pairs close to the prime vertical, the exact evaluation of the hour angles are not critical. A rough approximation of the clock correction is

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[6]Niethammer. op. cit.

good enough because of the minus sign in the numerator, as can be seen from the following.

Let ΔT be the tone clock correction, and τ the error in the estimate value ΔT_o, so

$$\Delta T = \Delta T_o + \tau$$

Hence, the hour angles will be

$$t_w = \theta_w - \alpha_w + \Delta T_o + \tau = B_w + \tau$$

$$t_e = \theta_e - \alpha_e + \Delta T_o + \tau = B_e + \tau$$

In the prime vertical we have

$$\tan \delta_e = \tan \theta \cos t_e$$

$$\tan \delta_w = \tan \theta \cos t_w$$

Replacing these values in the above equation and after some simplifications, we get

$$d\phi = \tau \frac{\cos B_w \cos t_e - \cos B_e \cos t_w}{2 \sin \sigma}$$

The numerator of this equation tends to zero as τ tends to zero. Hence, taken the value of τ below 14 seconds of time, there will be no error influence in the latitude. We run a test with a double purpose:

1. To evaluate the error $d\phi$ by assuming in the example that using a star pair according to ocular instrument position, $= \Delta T = - 24^{s}.8$.

Using $\tau = o$ to check the values of latitude according to the test conducted before.

The test conducted is shown in table 10.

Table 10. Tests results with and without ΔT

Star Pair	Star West	Star East	Latitude ΔT not included	Latitude $\Delta T = -24^{s}8$ included
1-2	N-S	S-N	47° 32' 29"30	47° 32' 29".49
3-4	S-N	N-S	47 32 25.91	47 32 26.06

Same Stars but in Reversed Order

1-3	N-S	N-S	47^0 32' 27.″50	47^0 32' 27.″66
2-4	S-N	S-N	47 32 27.59	47 32 27.75

It can be seen from table 10 that even an error

$$\tau = -24.^s8$$

produces an error in latitude

$$d\phi < 0''2$$

The results shown in table 10 are in complete agreement with the values shown in table 9.

Table 11. Evaluation of latitude with respect to selecting star pairs

Star	Ocular Position		Transit Time	Right Ascension	Declination
β Lira	W	N-S	22 20^m $40^s.381$	$18^h48^m03^s.382$	$33^018'09''.67$
β Triang	E	S-N	22 44 11.739	02 06 18.838	34 43 54.17
Boss 746	E	N-S	23 48 22.217	03 15 20.096	34 01 30.14
λ Cyg	W	S-N	23 56 48.368	20 45 17.254	36 17 37.94

		West (N-S) β Lyr	East (N-S) Boss 746	East (S-N) β Triang	West (S-N) λ Cyg
1	θ	$22^h20^m40^s.381$	$23^h48^m22.217$	22 44 12.739	$23^h56^m48.638$
2	$\Delta\mu(\theta_i - \theta_1)$	0	−0.042	−0.012	−0.045
3	$d\theta_k$	+0.274	+0.281	+0.288	+0.306
4	$d\theta$	+0.165	−0.203	−0.205	+0.278
5	α	18 48 03.382	3 15 20.096	2 06 18.838	20 45 17.254
6	1+2+3+4−5	3 32 37.438	20 33 02.157	20 37 53.972	3 11 31.923
7	σ	$6^h59^m25^s.282$		6^h33^m $37^s.951$	
8	$\frac{1}{2}P$	52^0 26'54''.62		49 12 14.63	
9	δ_w	33 18 09.67		36 17 37.94	
10	δ_e	34 01 30.14		34 43 54.17	
11	½(9+10)	33 39 49.90		35 30 41.06	
12	½(9−10)	−0 21 40.24		0 46 56.88	

$$\tan \tfrac{1}{2}(B+C) = \frac{\cos\ (12)}{\sin\ (11)} \cot\ (8) \qquad \tan \tfrac{1}{2}(B-C) = \frac{\sin\ (12)}{\cos\ (11)} \cot\ (8)$$

13	½(B+C)	$54^0$12' 20''.06	$56^0$03'16''.90
14	½(B−C)	−0 20 0.94	0 49 41.09
	B	53°52' 19''.12	56°52' 57''.99
	C	305 27 39.00	304 46 24.18

$$\tan \phi = \pm \frac{\sqrt{\cos^2 P + \tan^2 \delta}}{\sin P}$$

tan ϕ	1.0928 78331	1.0928 25492
ϕ	$47^0$32'27''.64	$47^0$32'27''.16
Niethammer gives	27.69	27.80
dif	0''.05	0''.04

Table 12. Evaluating latitude by reversing star order

		West (N-S)	East (S-N)	East (N-S)	West (S-N)
		β Lira	β Triang	Boss 746	λ Cyg
1	θ	$22^h20^m40^s.381$	$22^h44^m12^s.739$	$23^h48^m22^s.217$	$23^h56^m48^s.630$
2	$\Delta\mu(\theta_i-\theta_1)$	0	−0.012	−0.042	−0.045
3	$d\theta_k$	+0.274	+0.288	+0.281	+0.306
4	$d\theta_2$	+0.165	−0.205	−0.203	+0.278
5	α	18 48 03.382	2 06 18.838	3 15 20.096	20 45 17.254
6	1+2+3+4−5	3 32 37.439	20 37 53.972	20 33 02.157	3 11 31.923
7	σ_{12}	$6^h54^m\ 43^s.467$		$6^h38^m29^s.766$	
8	$\frac{1}{2}\rho^0$	$51^0 50'\ 26''.00$		$49^0 48'\ 43''.24$	
9	δ_w	33 18 09.67		36 17 37.94	
10	δ_e	34 43 54.17		34 01 30.14	
11	½(9+10)	34 01 01.92		35 09 34.04	
12	½(9−10)	−0 42 52.25		1 08 3.90	

$$\tan\tfrac{1}{2}(B+C)=\frac{\cos(12)}{\sin(11)}\operatorname{ctn}(8) \qquad \tan\tfrac{1}{2}(B-C)=\frac{\sin(12)}{\cos(11)}\operatorname{ctn}(8)$$

13	½(B+C)	$54^0 32'\ 54''.77$	$55^0 42' 42''.11$
14	½(B−C)	−0 40 38.34	1 10 18.68
15	$13+14=P_w$	$53^0 52'\ 16.43$	$56^0 52' 56''.85$
16	$13-14=-P_e$	304 46 28.14	305 27 36.56

$$\tan\phi = \pm\frac{\sqrt{\cos^2 P + \tan^2\delta}}{\sin P} \quad \begin{matrix} +\text{ west} \\ -\text{ east}\end{matrix}$$

17	$\tan\phi$	1.0928 38497	1.092861476
18	ϕ	$47^0 32' 29.47$	$47^0 32' 26.09$

Table 13. Testing accuracy in determining latitude

		ΔT not included		ΔT included	
		Star			
		East (N-S)	West (S-N)		
		β Lyr	β Triang		
1	θ	$22^h20^m40^s.381$	$22^h44^m12.739$		
2	$\Delta l+\mu(\theta_i\theta_0)$	0	-0.0012	-0.0012	-24.771
3	$d\theta_k$	0.274	0.288		
4	$d\theta$	+0.165	-0.205		
5	α	18 48 03.382	206 18.838		
6	1+2+3+4-5	$3^h32^m37^s.439$	$20^h37^m53^s.972$	3^h32 12.680	$20^h37^m29^s.213$
7	B=6x15	$53^009'21''.58$	$309^028'29''.58$	$53^003'10''.20$	$309^022'18''.20$
8	$\sigma = B_w-B_e$	$103^040'52''00$		$103^040'52''00$	
9	$\tan\delta e \sin B_w$	0.55478 9557		0.55404 0178	
10	$\tan\delta w\ B_e$	-0.50709 7530		-0.50784 8679	
11	9-10	1.06188 7087		1.0618857	
12	$\sin\sigma$	0.97162 7146			
13	11/12 $\tan\phi$	1.09289 5655		1.09289 7478	
	ϕ	$47^032'29''30$		$47^o32'29''.47$	

Table 14. Mean transit times and their differences for each vertical line

Vertical	East: Ocular Position N-S		West: Ocular Position S-N	
Line	$\theta_m = \frac{1}{2}(\theta_1+\theta_2)$	$\theta_2-\theta_1$	$\theta = \frac{1}{2}(\theta_1+\theta_2)$	$\theta_2-\theta_1$
6	$19^h36^m04^s.85$	$9^m28^s.83$	$20^h44^m57.87$	$9^m26^s.88$
5	35 56.59	7 05.35	45 06.77	7 02.68
4	35 50.85	4 48.23	45 11.77	4 46.28
3	35 46.89	2 26.75	45 15.52	2 23.78
	$i_1 = -3''.76$	$i_2 = -6''.90$	$i_1 = -7''.36$	$i_2 = -3''.72$

$$\theta_0 = \frac{1}{2}(\theta_1 + \theta_2) \pm \frac{1}{2}(I_2 + I_1) - \delta\theta + W$$

$$\tau = (\theta_2 + \theta_1) \pm (I_2 + I_1) \qquad -E$$

$$\delta\theta = 0^s.5256\ A \sin\delta\ (\tau^0)^2$$

$$I = \frac{1}{15} A \sin\delta \operatorname{cosec}\phi$$

$$A = \frac{1}{\sqrt{\cos^2\delta - \cos^2\phi}}$$

$$\delta = 46^0 26' 31''.93$$

$$\phi_0 = 46^0\ 46'\ 16''$$

Table 14 shows in the first column the vertical line of observation, in the second column the mean of transit times, in the third column the difference in the transit times all with respect to the east observation, in the fourth and fifth columns the data from the west observation. At the bottom of the table, the inclinations are shown that correspond to a direct and reversed instrument position. Table 15 is self explanatory.

Table 16 shows in the second column the correct values of τ after being corrected for inclination, in the third column the correction $\delta\theta$, and in the last column the reduced mean time. The mean time θ_m increases from line 3 to line 6, and $\delta\theta$ increases in the same way; therefore, $\delta\theta$ must be subtracted from the mean θ_m.

Table 17 gives the values for the west transit. Here the θm values shown in table 14 decreased from line 3 to 6, while the corrections $\delta\theta$ increased. Hence, $\delta\theta$ must be added to the θm times.

Table 18 shows in the last column the values of latitude as resulted for each line. Our evaluation shows a discrepancy of 0.15 second of arc with respect to Struve's method.

Table 15. Corrections owing to inclination errors

	I_1	I_2	$I_2 - I_1$	$\frac{1}{2}(I_2+I_1)$
East	$-3^s.293$	$-6^s.043$	$+2^s.750$	$-4^s.668$
West	-6.446	-3.258	+3.188	-4.852

Table 16. Reduction time to the center vertical line (east star)

			τ	$\delta\theta$	$\theta_0 = \theta_m + 4^s.668 - \delta\theta$
	6	9^m	$26^s.080$	$19^s.284$	$19^h 35^m 50^s.234$
East	6	7	02.600	10.782	50.476
	4	4	45.480	4.951	50.566
	3	2	24.000	1.283	50.275

Table 17. Reduction time to the center vertical line (west star)

			τ	$\delta\theta$	$\theta_0 = \theta_m - 4.852 - \delta\theta$
	6	9^m	$30^s.068$	$19^s.368$	$20^h 45^m 12^s.256$
West	5	7	05.868	10.809	12.727
	4	4	49.468	4.994	11.912
	3	2	26.968	1.287	11.955

$$\tan p_w = \operatorname{cosec} \delta_w \cot \tfrac{1}{2} \alpha\sigma_{12}$$

$$\sigma_{12} = 15\,(\theta_w - \theta_e)$$

$$\tan \theta = \operatorname{cosec} P \sqrt{\cos^2 P + \tan^2 \delta}$$

Table 18. Evaluation of latitude

	$\frac{1}{2}\sigma_{12}$	tan P	P	tan ϕ	ϕ
6	8°40'15"92	9.04835348	83°41'36.30"	1.06381275	46°46'15.64"
5	8 40 16.68	9.04806872	83 41 35.59	1.06381351	15.72
4	8 40 10.10	9.05006684	83 41 40.56	1.06380817	15.20
3	8 40 12.60	9.04932931	83 41 38.73	1.06381014	15.39

Mean value.......................ϕ = 46°46'15.49"

Struve's method................ϕ = 46 46 15.34

Discrepancy 0.15"

Testing Baldini's Method At Goddard Space Flight Center. A test of Baldini's method was conducted on site at the Goddard Space Flight Center, Greenbelt, MD. Observations were performed by Rudolph Salvermoser, DMATC, assisted by William Allen, ETL. Each star pair was observed without reversing the instrument position and other two, in reverse instrument position. The collimation error was unknown; therefore, it was necessary to consider its determination also. *The latitude was computed through the* equation

$$\tan\phi = \frac{\sqrt{\cos^2 P_w + \tan^2\delta}}{\sin P_w} \tag{53}$$

and the parallactic angle from

$$\cot\text{an}\, P_w = \frac{\cos\delta_w \tan\delta_e}{\sin\sigma} - \sin\delta_w \cot\sigma \tag{54}$$

where

$$\alpha = (1 + x)(T_w - T_e) - (\alpha_w - \alpha_e) + c(A_w + A_e) + i_w B_w + i_e B_e \tag{55}$$

$1 + x$ = constant to convert mean time to sidereal time

c = collimation error

A_w, A_e = Coefficients of collimation of west and east star

B_w, B_e = Coefficients of inclination of west and east star

The coefficients A and B are

$$A = \frac{1}{\cos\delta\cos P} \qquad B = \frac{\cos z}{\cos\delta\cos P} = \frac{\tan\delta}{\sin\ell\cos P}$$

$$\cos Z = \sin\delta/\sin\phi$$

Let

$$\sigma_o = (1+x)(T_w-T_e) - (\alpha_w-\alpha_e) \tag{56}$$

$$d\sigma = c(A_w+A_e) + i_wB_w + i_eB_e \tag{57}$$

$$\sigma = \sigma_o + d\sigma \tag{58}$$

Let ϕo be the latitude when σ_o is used instead of considering σ; then,

$$\phi = \phi_o + d\phi \tag{59}$$

To obtain $d\phi$, we differentiate equation (53), we have

$$d\phi = -\frac{\cot P_w}{\tan\phi}\, dP \tag{60}$$

dP is obtained through equation (54). We obtain by differentiation

$$dP = -\frac{\cos P_e \sin P_w}{\sin \sigma} d\sigma \tag{61}$$

whence

$$d\phi = +\frac{\cos P_w \cos P_e}{\sin \sigma \tan \phi} d\sigma$$

Replacing $d\sigma$ with the values given by equation (57), we obtain an equation of condition of

$$d\phi = +\frac{\cos P_w \cos P_e}{\tan \phi \sin \sigma} \left[c(A_w+A_e)+i_w B_w+i_e B_e\right] \tag{62}$$

Hence, equation (59) becomes

$$\phi = \phi_{oi} \pm R_i (A_w + A_e)_i c + R_i \, (B_w + B_e)_i \tag{63}$$

The $\pm$ sign depends whether the observations are made up with the instrument in the direct or the reverse position;

$$i = \text{number of the star pair}$$

$$R = \frac{\cos P_w \cos P_e}{\tan \phi \sin \sigma}$$

PRIME VERTICAL TRANSIT METHOD LATITUDE OBSERVATIONS

STAR #	UT OF TRANSIT			
3388	00	16	55.959	W
0066	00	23	14.210	E
0045	00	33	55.499	E
3457	00	40	12.177	W
1488	00	50	31.446	W
0018	00	54	39.415	E
2072	01	06	06.961	E
3518	01	08	37.149	W

Assumed Latitude: 39^{o} ol' 15" N

Assumed Longitude: $05^{h}07^{m}$ 18.9^{s} W

Date of Observ'n 29 Oct 1981 GCD

Time Signal Station WWV

Prel. Signal Corr'n + 146^{s}

Note: Chron is 5 ms

Faster than UT signal

$\frac{1}{2}$ R(m+s).200 = 0.041^{s} = $0\overset{''}{.}62$

Level Vial Constant = $+0\overset{''}{.}9754 - 0\overset{''}{.}009267(^{0}F) + 0\overset{''}{.}0001374(^{0}F^{2})$

1 Revolution of the drum is 9.51^{s}(Tan Q) (Sec Lat)

Std. Dev. of one "tick" = $\pm 0\overset{s}{.}068$ tan Q sec ϕ

GCD=29 Oct 81 STA. AIP Astro

Star #	R.A.			δ		
3388	17^h	26^m	$00^s.097$	20^0	06'	00".183
66	01	53	$38^s.272$	20	43	10".271
45	01	18	$28^s.195$	27	10	09".088
3457	18	18	24.076	24	26	32.227
1488	18	45	19.007	26	38	47".158
18	00	35	54.790	33	37	13".430
2072	01	07	01.162	31	54	57".296
3518	19	00	31.606	26	16	09".649

Table 19. Evaluating latitude, collimation, and inclination coefficients, forming an equation of condition

Inst. Pos.	Star	Latitude ϕ_0	West A_w	West B_w	East A_e	East B_e	Coefficient of Reduction R
D	3388	39°01'11".15	1.896	1.035	1.920	1.079	0.473
D	66						
R	45	39 01 18.46	-2.107	1.385	-2.307	1.673	0.327
R	3457						
R	1488	39 01 17.22	-2.263	1.612	-3.337	2.934	0.220
R	18						
D	2072	39 01 11.89	2.233	1.570	2.925	2.456	0.248
D	3518						

Equation of Condition

$$\phi = \phi_0 + (A_w + A_e)\ R \cdot c + (B_w\ i_w + B_e\ i_e)R$$

Table 20. **Mean Level Readings and Inclination Corrections**

Pair	Inst. Pos	Eyepiece	Time	L_s	L_n	Lo	i
1	D	South	0^h 20	48.55		49.36	-0"79
2	R	North	0 37		50.18		-0.79
3	R	North	0 52		49.92		+0.10
4	D	South	1 08	50.12		50.02	+0.10

Equation of Condition

$$\phi = \phi_0 + c(A_w + A_e) + (i_w B_w + i_e B_e)R$$

Pair 1 $\quad \phi = 39^0 01' 11''15 + 1.806\ c + i_1$

Pair 2 $\quad \phi = \quad 18''46 - 1.443\ c + i_1$

Pair 3 $\quad \phi = \quad 11.89 + 1.281\ c + i_2$

Pair 4 $\quad \phi = 39\ 01\ \ 17.22 - 1.232C + i_2$

$$(1+4) - (2+3) \qquad 12''64 - 5.762c = 0$$

$$c = + 2''\ 194$$

With this value of collimation and the values of inclination from table 1, we obtain from equations (1) through (9) the final values,

Pair	Latitude
1	39°01'14.32"
2	14.50
3	14.80
4	39 01 14.62
	Mean ϕ = 39°01'14.56"

Comments on Niethammer's Prime Vertical Latitude Determination. Neithammer's method for determining latitude from observations of a star pair in the prime vertical is based in deriving the exact star hour angle over the prime vertical plane. Although it may be used for the stated purpose, it is not straightforward.

The latitude is computed through the equation

$$\tan \phi = \frac{\tan \sigma w}{\cos(tw-\mu_n)} = \frac{\tan \delta e}{\cos(te+\mu_n)}$$

This equation is equivalent to the equation

$$\tan d = \frac{\tan w}{\cos t_v}$$

being t_v, the star hour angle in the prime vertical. An error in determining the hour angle and the value of μ_n will reflect in an error upon the latitude as follows:

$$d\phi = - \frac{\tan \phi}{1+\tan 2\phi} \tan(t_w-\mu)(dt_w-d\mu_n)$$

The hour angle t_w is the function of the clock correction, and μ is the function of azimuth; therefore, it must fulfill the following requirements:

a. To determine the exact value of the clock correction.

b. The azimuth of the northern end of the rotation axis, taken position from north to east.

c. The instrument must be brought to within a few seconds of arc with respect to the prime vertical.

To fulfill these requirements, extra field work is required. Hence, to ensure a higher accuracy, good methods must be used in determining the evaluation of the constants above mentioned, which represent a clear disadvantage.

Conclusion

The Baldini equations are less complicated to apply than those associated with Niethammer's method, and they can lead to more accurate results since their application is more straightforward and involves fewer potential sources of error.

END

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